Dynamics of entanglement for a two-parameter class of states in a qubit-qutrit system[*]

Hai-Rui Wei, Bao-Cang Ren, Tao Li, Ming Hua, and Fu-Guo Deng[†]

Department of Physics, Applied Optics Beijing Area Major Laboratory,
Beijing Normal University, Beijing 100875, China
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We investigate the dynamics of entanglement for a two-parameter class of states in a hybrid qubit-qutrit system under the influence of various dissipative channels. Our results show that entanglement sudden death (ESD) is a general phenomenon and it usually takes place in a qubit-qutrit system interacting with various noisy channels, not only the case with dephasing and depolarizing channels observed by others. ESD can only be avoided for some initially entangled states under some particular noisy channels. Moreover, the environment affects the entanglement and the coherence of the system in very different ways.

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I. INTRODUCTION

Quantum entanglement is a vital resource for quantum information processing [1]. However, isolating a quantum system completely from its environment is plainly an impossible task and each quantum system will inevitably interact with its environment. Therefore, it is important to investigate the behavior of an entangled quantum system under the influence of its environment. Recently, Yu and Eberly [2, 3] investigated the dynamics of two-qubit entangled states undergoing various modes of decoherence. They found that it takes an infinite time to complete decoherence locally, the global entanglement may be lost in a finite time, and the decay of a single-qubit coherence can be slower than the decay of a two-qubit entanglement. The abrupt disappearance of entanglement in a finite time was named "entanglement sudden death"(ESD). A geometric interpretation of the phenomenon is given in Ref.[4]. In addition, experimental evidences of ESD have been reported for optical setups [5, 6] and atomic ensembles [7]. Clearly, ESD can seriously affect the applications of entangled states in a practical quantum information processing. Recently, dynamics of entanglement has received increasing attention [8–10].

ESD in finite-dimensional systems is not limited only to two-qubit systems. It may be occurs in a composite quantum system with a larger dimension and a multi-qubit system as well [11–17]. The dissipative dynamics for a specific one-parameter class of states in a qubit-qutrit ($2 \otimes 3$) system interacting with dephasing and depolarizing channels was studied by Ann *et al.* [13] and Khan [14], respectively. Ann *et al.* [13] conjectured that ESD exists in all bipartite quantum systems. Khan [14] showed that no ESD happens in any density matrix of a qubit-qutrit system when only the qubit is coupled to its local depolarizing channel but the re-birth of entanglement occurs in particular initial states. However, for general qubit-qutrit states and other common noise channels, the dissipative dynamics of a hybrid qubit-qutrit is not presented.

In this paper, we devote to investigate the behavior of entanglement for a two-parameter class of states in a qubit-qutrit system under the influence of both two independent (multi-local) and only one (local) various noise channels, such as dephasing, phase-flip, bit-(trit-) flip, bit-(trit-) phase-flip, and depolarizing channels. Using negativity for quantifying entanglement, some analytical or numerical results are presented. We find that ESD is a general phenomenon in a qubit-qutrit system undergoing all these noise channels, not only the case with dephasing and depolarizing channels observed by others [13]. It is interesting to show that ESD always takes place in any density matrix when each subsystem couples to its depolarizing channel or only the qutrit couples to its trit-flip or trit-phase-flip channels. ESD can only be avoided in some initial states undergoing particular noise channels. For example, no ESD occurs when the system under the influence of multi-local (local) dephasing, multi-local (local) phase-flip, local bit-flip, and local bit-phase-flip channels if it is initially in the state shown in Eq.(1) with the parameter $b = 0$. Our results show that the noise channels affect the entanglement and the coherence of a hybrid qubit-qutrit system in very different ways. For local or multi-local dephasing, phase-flip, and depolarizing noise channels, a time scale of disentanglement is usually shorter than the decay of the off-diagonal dynamics, and coherence disappears in an infinite-time limit. For multi-local and local bit-flip and bit-phase-flip channels, disentanglement occurs in an infinite time, but coherence does not disappear even though $t \mapsto \propto$.

This paper is organized as follows. In Sec.II, we motivate the choice of a two-parameter class of states in a qubit-qutrit system, and the physical model are introduced. In Sec.III, entanglement dynamics of a two-parameter class of states in a qubit-qutrit system under the influence of local and multi-local dephasing, phase-flip, bit-(trit-) flip, bit-(trit-) phase-flip, and depolarizing noise channels are discussed, respectively. Discussion and

summary are shown in Sec.IV.

II.   INITIAL STATES AND NOISE MODEL

A two-parameter class of states with real parameters in a hybrid qubit-qutrit $(2 \otimes 3)$ quantum system [18] can be described as

$$
\begin{aligned}
\rho_{bc}(0) \ = \ & a\left(|02\rangle\langle 02| + |12\rangle\langle 12|\right) + b(|\phi^+\rangle\langle\phi^+| + \\
& |\phi^-\rangle\langle\phi^-| + |\psi^+\rangle\langle\psi^+|) + c|\psi^-\rangle\langle\psi^-|, \quad (1)
\end{aligned}
$$

where

$$
\begin{aligned}
|\phi^\pm\rangle \ &= \ \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \\
|\psi^\pm\rangle \ &= \ \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \quad (2)
\end{aligned}
$$

and a, b, and c are three real parameters, and they satisfy the relation $2a + 3b + c = 1$. $|0\rangle$ and $|1\rangle$ are the two eigenstates of a two-level quantum system (qubit) or the eigenstates of a three-level quantum system (qutrit) with the other eigenstate $|2\rangle$. The two-parameter class of states $\rho_{bc}(0)$ can be obtained from an arbitrary state of a $2 \otimes 3$ quantum system by means of local quantum operations and classical communication [18].

For an arbitrary mixed state ρ^{AB} in a $2 \otimes 2$ or a $2 \otimes 3$ system, its entanglement can well be characterized and quantified by its negativity,[19, 20]

$$
N(\rho^{AB}) = \| \rho^{T_B} \|_1 - 1, \quad (3)
$$

which corresponds to the absolute value of the sum of negative eigenvalues of ρ^{T_B} (the partial transpose ρ^{T_B} associated with an arbitrary product orthonormal basis $f_i \otimes f_j$ is defined by the matrix elements: $\rho_{m\mu,n\nu}^{T_B} \equiv \langle f_m \otimes f_\mu | \rho^{T_B} | f_n \otimes f_\nu \rangle = \rho_{m\nu,n\mu}$), i.e.,

$$
N(\rho^{AB}) = 2 \max\{0, -\lambda_S\}, \quad (4)
$$

where λ_S represents the sum of all negative eigenvalues of ρ^{T_B}. $N(\rho^{AB}) = 0$ for an unentangled state. Therefore, from Eq.(1) one can obtain the range of parameters as $3b < c \leq 1 - 3b$, i.e., $b \in [0, 1/6]$ for the initial entangled states.

In our physical model of noise for a qubit-qutrit system (composed of a two-level subsystem A and a three-level subsystem B), the two subsystems interact with their environments independently. The evolved states of the initial density matrix of such a system when it is influenced by multi-local environments can be given compactly by

$$
\rho_{bc}^{AB}(t) = \sum_{i=1}^{2} \sum_{j=1}^{3} F_j^B E_i^A \rho_{bc}^{AB}(0) E_i^{A\dagger} F_j^{B\dagger}. \quad (5)
$$

Here, the operators E_i^A and F_j^B are the Kraus operators which are used to describe the noise channels acting on the qubit A and the qutrit B, respectively. They satisfy the completeness relations $E_i^{A\dagger} E_i^A = I$ and $F_j^{B\dagger} F_j^B = I$ for all t.

III.   DYNAMICS OF ENTANGLEMENT UNDER DECOHERENCE

It is important to consider the possible degradation of any initially prepared entanglement due to decoherence. In this section we investigate what happens to the entanglement in a qubit-qutrit system under common noise channels for qubit (qutrit): dephasing, phase-flip, bit-(trit-) flip, bit-(trit-) phase-flip, and depolarizing channels. The two specific environment noise situations will be considered: (i) local and (ii) multi-local. In the case (i), only one part of a qubit-qutrit system (S) interacts with its environment. In the case (ii), both the two parts of S interact with their local environments, independently.

A.   Dephasing channels

The set of Kraus operators for a single qubit A and a single qutrit B that reproduce the effect of a dephasing channel are given by

$$
\begin{aligned}
E_1^A \ &= \ \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma_A} \end{pmatrix} \otimes I_3, \\
E_2^A \ &= \ \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\gamma_A} \end{pmatrix} \otimes I_3, \quad (6) \\
F_1^B \ &= \ I_2 \otimes \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\gamma_B} & 0 \\ 0 & 0 & \sqrt{1-\gamma_B} \end{pmatrix}, \\
F_2^B \ &= \ I_2 \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{\gamma_B} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
F_3^B \ &= \ I_2 \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{\gamma_B} \end{pmatrix}. \quad (7)
\end{aligned}
$$

The time-dependent parameters are defined as $\gamma_A = 1 - e^{-t\Gamma_A}$ and $\gamma_B = 1 - e^{-t\Gamma_B}$. Here $\gamma_A, \gamma_B \in [0, 1]$. Γ_A (Γ_B) denotes the decay rate of the subsystem A (B).

According to Eq.(5), the time-dependent evolved density operator $\rho^{AB}(t)$ of a hybrid qubit-qutrit system, which is initially in the entangled state $\rho^{AB}(0)$, is given by

$$\rho^{AB}(t) = \begin{pmatrix} b & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{b+c}{2} & 0 & \frac{(b-c)\sqrt{(1-\gamma_A)(1-\gamma_B)}}{2} & 0 & 0 \\ 0 & 0 & \frac{1-c-3b}{2} & 0 & 0 & 0 \\ 0 & \frac{(b-c)\sqrt{(1-\gamma_A)(1-\gamma_B)}}{2} & 0 & \frac{b+c}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-c-3b}{2} \end{pmatrix}. \tag{8}$$

In order to characterize the dynamics of evolution for the density matrix and consider the entanglement of this system quantified by negativity, we should calculate the eigenvalues of the partial transpose of the time-evolved density matrix $\rho^{AB}(t)$ and determine the potential negative eigenvalues.

(1) Multi-local dephasing channel. The eigenvalue which can potentially be negative is

$$\lambda = b - \frac{c-b}{2}\sqrt{(1-\gamma_A)(1-\gamma_B)}. \tag{9}$$

The entanglement of the qubit-qutrit system under a multi-local dephasing channel is

$$N^{mul-loc}(\rho^{AB}) = 2\max\{0, \frac{c-b}{2}\sqrt{(1-\gamma_A)(1-\gamma_B)} - b\}. \tag{10}$$

It is easy to obtain that all the states which are initially entangled ($3b < c \leq 1-3b$) become separable when $(1-\gamma_A)(1-\gamma_B) \leq \left(\frac{2b}{c-b}\right)^2$ and $b \neq 0$.

(2) Qubit dephasing channel only. If the qutrit field is turned off (i.e., $\gamma_B = 0$), that is, only a dephasing noise acts on qubit A alone, the entanglement of the qubit-qutrit system is

$$N^{bit}(\rho^{AB}) = 2\max\{0, \frac{c-b}{2}\sqrt{1-\gamma_A} - b\}. \tag{11}$$

All the states which are initially entangled ($3b < c \leq 1-3b$) become separable as soon as $\gamma_A \geq 1 - \left(\frac{2b}{c-b}\right)^2$ and $b \neq 0$.

(3) Qutrit dephasing channel only. By the same argument as that made in the qubit dephasing noise, for a dephasing noise acting on qutrit B alone, the entanglement is

$$N^{trit}(\rho^{AB}) = 2\max\{0, \frac{c-b}{2}\sqrt{1-\gamma_B} - b\}. \tag{12}$$

A single-qutrit dephasing channel will induce ESD in the qubit-qutrit system when $\gamma_B \geq 1 - \left(\frac{2b}{c-b}\right)^2$ and $b \neq 0$.

Together with these pieces of results, one can see that ESD takes place under local and multi-local dephasing channels in a general qubit-qutrit system if and only if the system is initially in the state shown in Eq.(1) with the parameter $b \neq 0$. Moreover, the smaller c/b, the longer the death time range. However, if $b = 0$, no ESD occurs.

B. Phase-flip channels

The Kraus operators describing the phase-flip channel for a single qubit A are given by

$$E_1^A = \sqrt{1 - \frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_3,$$

$$E_2^A = \sqrt{\frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes I_3, \tag{13}$$

and those for a single qutrit B can be written as

$$F_1^B = I_2 \otimes \sqrt{1 - \frac{2\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$F_2^B = I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i2\pi/3} & 0 \\ 0 & 0 & e^{i2\pi/3} \end{pmatrix},$$

$$F_3^B = I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \end{pmatrix}, \tag{14}$$

where $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, and $\gamma_A, \gamma_B \in [0,1]$. Γ_A (Γ_B) represents the decay rate of the subsystem A (B).

We can obtain the time-evolved density-matrix dynamics $\rho^{AB}(t)$ of the qubit-qutrit system under a phase-flip channel, according to Eq.(5). That is,

$$\rho^{AB}(t) = \begin{pmatrix} b & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{b+c}{2} & 0 & \frac{(b-c)(1-\gamma_A)(1-\gamma_B)}{2} & 0 & 0 \\ 0 & 0 & \frac{1-c-3b}{2} & 0 & 0 & 0 \\ 0 & \frac{(b-c)(1-\gamma_A)(1-\gamma_B)}{2} & 0 & \frac{b+c}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-c-3b}{2} \end{pmatrix}. \tag{15}$$

With the same argument as that made in dephasing channels, some results can be presented as follows.

(1) Multi-local phase-flip channel. The entanglement of the qubit-qutrit system under a multi-local phase-flip channel is calculated as

$$N^{mul-loc}(\rho^{AB}) = 2\max\left\{0, \frac{(c-3b)+(c-b)((1-\gamma_A)(1-\gamma_B)-1)}{2}\right\}.$$

That is, all the states which are initially entangled ($3b < c \leq 1-3b$) become separable for all values of $(1-\gamma_A)(1-\gamma_B) \leq \frac{2b}{c-b}$ and $b \neq 0$.

(2) Qubit phase-flip channel only. The entanglement of the system is

$$N^{bit}(\rho^{AB}) = 2\max\{0, \frac{c-3b-\gamma_A(c-b)}{2}\}. \tag{16}$$

All the states which are initially entangled ($3b < c \leq 1-3b$) become separable as soon as $\gamma_A \geq \frac{c-3b}{c-b}$ and $b \neq 0$.

(3) Qutrit phase-flip channel only. The negativity for all the states which are initially entangled ($3b < c \leq 1-3b$) is given as

$$N^{trit}(\rho^{AB}) = 2\max\{0, \frac{c-3b-\gamma_B(c-b)}{2}\}. \tag{17}$$

All the states which are initially entangled become separable when $\gamma_B \geq \frac{c-3b}{c-b}$ and $b \neq 0$.

Similar to the case with a dephasing noise, ESD also takes place when the system undergos a phase-flip noise environment but no ESD occurs when the parameter $b = 0$. Difference from the dephasing channel, the decay time range under the multi-local phase-flip channel is shorter than that in the local channel if $3b < c < 5b$, and longer for $5b < c \leq 1-3b$.

C. Bit-(Trit-) flip channels

The Kraus operators describing the bit-flip channel for a single qubit A are given by

$$\begin{aligned} E_1^A &= \sqrt{1-\frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_3, \\ E_2^A &= \sqrt{\frac{\gamma_A}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes I_3, \end{aligned} \tag{18}$$

and those for a single qutrit B can be written as

$$\begin{aligned} F_1^B &= I_2 \otimes \sqrt{1-\frac{2\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ F_2^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\ F_3^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{19}$$

where $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, and $\gamma_A, \gamma_B \in [0,1]$.

The matrix elements of ρ after the interaction time t under a multi-local bit-(trit-) flip channel are given by

$$\begin{aligned} \rho_{11}(t) &= \rho_{55}(t) = \frac{1}{12}(12b + 3(b-c)\gamma_A(\gamma_B-1) \\ &\quad + 2(1-6b)\gamma_B), \\ \rho_{22}(t) &= \rho_{44}(t) = \frac{1}{12}(6(b+c) - 3(b-c)\gamma_A(\gamma_B-1) \end{aligned}$$

$$+(2-6b-6c)\gamma_B),$$

$$\rho_{33}(t) = \rho_{66}(t) = \frac{1}{6}\left(3(1-3b-c)+(9b+3c-2)\gamma_B\right),$$

$$\rho_{15}(t) = \rho_{51}(t) = \frac{1}{12}(b-c)\gamma_A(3-2\gamma_B),$$

$$\rho_{35}(t) = \rho_{53}(t) = \rho_{16}(t) = \rho_{61}(t) = \frac{1}{12}(b-c)(2-\gamma_A)\gamma_B,$$

$$\rho_{26}(t) = \rho_{62}(t) = \rho_{34}(t) = \rho_{43}(t) = \frac{1}{12}(b-c)\gamma_A\gamma_B,$$

$$\rho_{24}(t) = \rho_{42}(t) = \frac{1}{12}(b-c)(2-\gamma_A)(3-2\gamma_B), \tag{20}$$

and all the remaining matrix elements are zero. Here $\rho = (\rho_{ij})$ and $i,j = 1,\ldots,6$.

(1) Qubit bit-flip channel only. We obtain the same results as the case with a phase-flip channel.

(2) Qutrit trit-flip channel only. The negativity for all the states which are initially entangled $(3b < c \leq 1 - 3b)$ can be given by

$$N^{trit}(\rho^{AB}) = 2\max\left\{0, \frac{3b-9c-(1-8b+2c)\gamma_B}{6}\right\}. \tag{21}$$

The states become separable once $\gamma_B \geq \frac{3c-9b}{1-8b+2c}$.

(3) Multi-local bit-(trit-) flip channel. For simplicity, we choose the local asympotic bit-(trit-) flip rates $\Gamma_A = \Gamma_B = \Gamma$ to discuss the effect of this noise on entanglement. Unfortunately, it is difficult to calculate the analytical eigenvalues of the partial transpose of the time-evolved density matrix. Therefore, we take the numerical calculation for various initial states as examples to investigate the dynamics of entanglement of the qubit-qutrit system under a multi-local bit-(trit-) flip channel.

As the first example, we consider the initial entangled states shown in Eq.(1) with the parameter $a = 0$ which are equivalent to Werner states [21] in a $2 \otimes 2$ systems but with different noise channels. Their dynamics of entanglement is shown in Fig.1. We note that the time-evolved density matrix for the initial state with the parameters $a = b = 0$ is just the case for the maximally entangled Bell state $|\psi^-\rangle$ [22], and the system does not suffer from ESD under a multi-local bit-(trit-) flip channel.

As a second example, we consider the initial entangled states with the parameter $b = 0$. Their dynamics of entanglement is shown in Fig.2.

Finally, we consider the initial states with $a, b, c > 0$. The dynamics of entanglement is plotted in Fig.3 for a particular value of the parameter $b = 0.05$.

From the above analysis and numerical results, we conjecture that ESD in a hybrid qubit-qutrit system under local and multi-local bit- (trit-) flip channels is a general phenomenon. No ESD takes place under a qubit-flip channel alone if and only if the qubit-qutrit system is initially in the states shown in Eq.(1) with the parameter
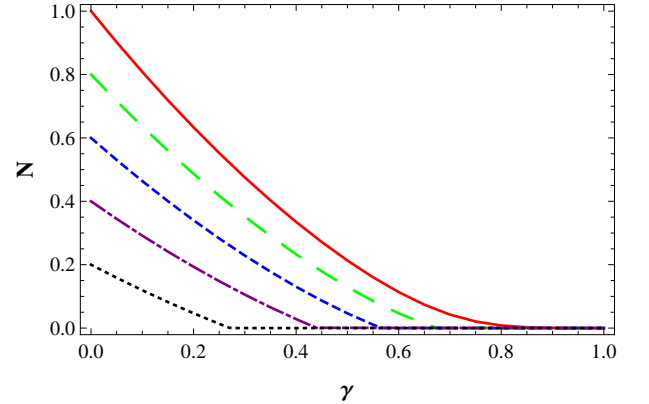

FIG. 1: Dynamics of entanglement for the system undergoing the multi-local bit- (trit-) flip noise with $a = 0$. N represents the negativity of a hybrid qubit-qutrit system. γ is the time-dependent parameter and $\gamma_A = \gamma_B = \gamma = 1 - e^{-t\Gamma}$. The solid, dashed, short-dashed, dashed-dotted, and dotted lines correspond to $b = 0$, $b = 1/30$, $b = 2/30$, $b = 3/30$, and $b = 4/30$, respectively.

$b = 0$. However, ESD always occurs when the system undergoes a qutrit-flip channel alone.
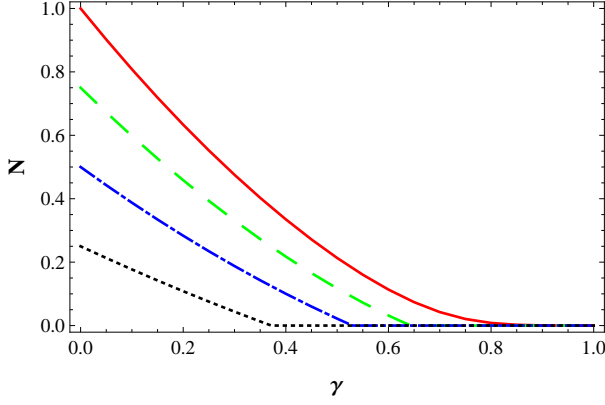
FIG. 2: Dynamics of entanglement for the system undergoing the multi-local bit-(trit-) flip noise with parameter $b = 0$. The solid, dashed, dashed-dotted, and dotted lines correspond to $c = 1$, $c = 3/4$, $c = 1/2$, and $c = 1/4$, respectively.
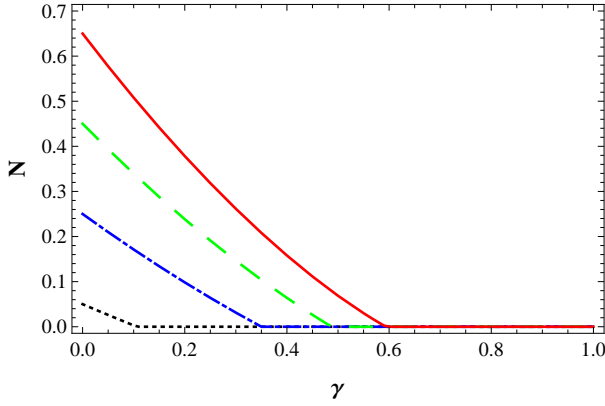

FIG. 3: Dynamics of entanglement for the system undergoing the multi-local bit-(trit-) flip noise with $b = 1/20$. The solid, dashed, dashed-dotted, and dotted lines correspond to $c = 16/20$, $c = 12/20$, $c = 8/20$, and $c = 4/20$, respectively.

D. Bit-(Trit-) phase-flip channels

The Kraus operators describing the bit-phase flip channel for a single qubit A are given by

$$
\begin{aligned}
E_1^A &= \sqrt{1 - \frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_3, \\
E_2^A &= \sqrt{\frac{\gamma_A}{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes I_3,
\end{aligned} \tag{22}
$$

and those for a single qutrit B can be written as

$$
\begin{aligned}
F_1^B &= I_2 \otimes \sqrt{1 - \frac{2\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
F_2^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & 0 & e^{i2\pi/3} \\ 1 & 0 & 0 \\ 0 & e^{-i2\pi/3} & 0 \end{pmatrix}, \\
F_3^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & 0 & e^{-i2\pi/3} \\ 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \end{pmatrix}, \\
F_4^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & e^{-i2\pi/3} & 0 \\ 0 & 0 & e^{i2\pi/3} \\ 1 & 0 & 0 \end{pmatrix}, \\
F_5^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \\ 1 & 0 & 0 \end{pmatrix}, \tag{23}
\end{aligned}
$$

where $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, and $\gamma_A, \gamma_B \in [0, 1]$.

The elements of the density matrix ρ after the interaction time t under multi-local bit-(trit-) phase-flip channels are given by

$$
\begin{aligned}
\rho_{11}(t) &= \rho_{55}(t) = b + \frac{1}{4}(b - c)\gamma_A(\gamma_B - 1) + (\frac{1}{6} - b)\gamma_B, \\
\rho_{22}(t) &= \rho_{44}(t) = \frac{1}{12}(6(b + c) - 3(b - c)\gamma_A(\gamma_B - 1) + (2 - 6b - 6c)\gamma_B), \\
\rho_{33}(t) &= \rho_{66}(t) = \frac{1}{6}(3(1 - 3b - c) + (9b + 3c - 2)\gamma_B), \\
\rho_{15}(t) &= \rho_{51}(t) = -\frac{1}{12}(b - c)\gamma_A(3 - 2\gamma_B), \\
\rho_{16}(t) &= \rho_{61}(t) = \rho_{35}(t) = \rho_{53}(t) = \frac{1}{24}(b - c)(\gamma_A - 2)\gamma_B, \\
\rho_{24}(t) &= \rho_{42}(t) = \frac{1}{12}(b - c)(2 - \gamma_A)(3 - 2\gamma_B), \\
\rho_{26}(t) &= \rho_{62}(t) = \rho_{34}(t) = \rho_{43}(t) = \frac{1}{12}(b - c)\gamma_A\gamma_B, \tag{24}
\end{aligned}
$$

and all the remaining matrix elements are zero.

(1) Local bit-(trit-) phase-flip channel only. We obtain the same results as the case with a bit-(trit-) flip channel.
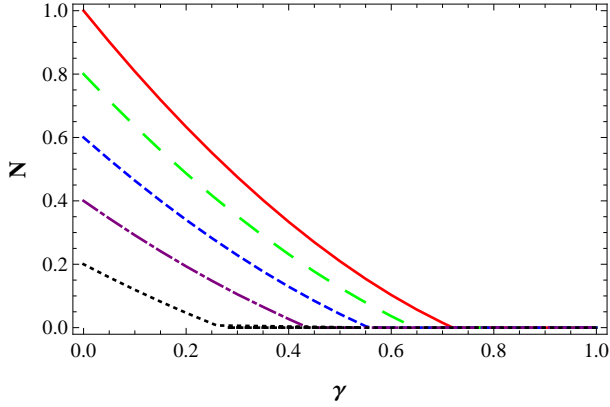

FIG. 4: Dynamics of entanglement for the system undergoing the multi-local bit-(trit-) phase-flip noise with the parameter $a = 0$. The solid, dashed, short-dashed, dashed-dotted, and dotted lines correspond to $b = 0$, $b = 1/30$, $b = 2/30$, $b = 3/30$, and $b = 4/30$, respectively.
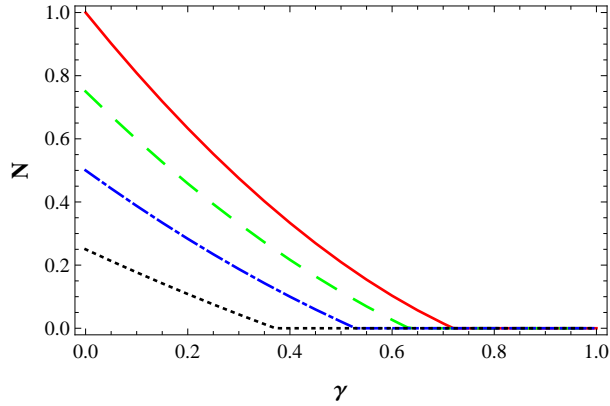

FIG. 5: Dynamics of entanglement for the system undergoing the multi-local bit-(trit-) phase-flip noise with the parameter $b = 0$. The solid, dashed, dashed-dotted, and dotted lines correspond to $c = 1$, $c = 3/4$, $c = 1/2$, and $c = 1/4$, respectively.

(2) Multi-local bit-(trit-) phase-flip channel. For simplicity, the time dependent parameters are also defined as $\gamma_A = \gamma_B = \gamma$. It is difficult to obtain the analytical results. The similar work is made as that in the case with a bit-(trit-) flip channel, and the dynamics of entanglement of the initial states with the parameter $a = 0$ is displayed in Fig.4. Different from a multi-local bit-(trit-) flip channel, one can see that there exists ESD for the maximally entangled Bell state (solid line). The dynamics of entanglement of the initial states shown in Eq.(1) with the parameters $b = 0$ and $a, b, c > 0$ are displayed in Fig.5 and Fig.6, respectively.
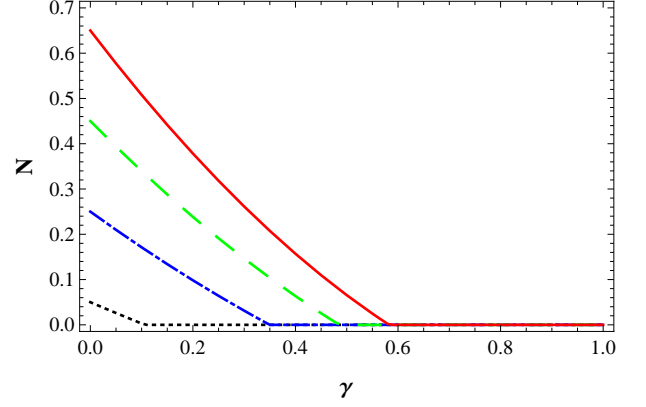

FIG. 6: Dynamics of entanglement for the system undergoing the multi-local bit-(tri-t) phase-flip noise with the parameter $b = 1/20$. The solid, dashed, dashed-dotted, and dotted lines correspond to $c = 16/20$, $c = 12/20$, $c = 8/20$, and $c = 4/20$, respectively.

E. Depolarizing channels

A depolarizing channel represents the process in which the density matrix is dynamically replaced by the maximally mixed state I/d. Here I is the identity matrix of a single qudit. The set of Kraus operators that reproduces the effect of the depolarizing channel for a single qubit A are given by

$$
\begin{aligned}
E_1^A &= \sqrt{1 - \frac{3\gamma_A}{4}} I_6, \\
E_2^A &= \sqrt{\frac{\gamma_A}{4}} \sigma_1 \otimes I_3, \\
E_3^A &= \sqrt{\frac{\gamma_A}{4}} \sigma_2 \otimes I_3, \\
E_4^A &= \sqrt{\frac{\gamma_A}{4}} \sigma_3 \otimes I_3, \quad (25)
\end{aligned}
$$

where σ_i $(i = 1, 2, 3)$ are the three Pauli matrices. The Kraus operators describing a single-qutrit depolarizing noise are given by[?]

$$
\begin{aligned}
F_1^B &= I_2 \otimes \sqrt{1 - \frac{8\gamma_B}{9}} I_3, \\
F_2^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y, \\
F_3^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Z, \\
F_4^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y^2, \\
F_5^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} YZ, \\
F_6^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y^2 Z,
\end{aligned}
$$

$$F_7^B = I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y Z^2,$$

$$F_8^B = I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y^2 Z^2,$$

$$F_9^B = I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Z^2, \qquad (26)$$

where

$$Y = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

$$Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \end{pmatrix}, \qquad (27)$$

and $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, $\gamma_A, \gamma_B \in [0, 1]$.

The matrix elements of ρ after the interaction time t under a multi-local depolarizing channel are given by

$$\rho_{11}(t) = \rho_{55}(t) = \frac{1}{12}(12b + 3(b - c)(\gamma_B - 1)\gamma_A + 2(1 - 6b)\gamma_B),$$

$$\rho_{22}(t) = \rho_{44}(t) = \frac{1}{12}(6(b + c) - 3(b - c)(\gamma_B - 1)\gamma_A + (2 - 6b - 6c)\gamma_B),$$

$$\rho_{33}(t) = \rho_{66}(t) = \frac{1}{6}(3(1 - 3b - c) + (9b + 3c - 2)\gamma_B),$$

$$\rho_{24}(t) = \rho_{42}(t) = \frac{1}{2}(b - c)(1 - \gamma_A)(-\gamma_B), \qquad (28)$$

and all the remaining matrix elements are zero.

(1) Multi-local depolarizing channel. The negativity for the composite system with initial entangled states $(3b < c \leq 1 - 3b)$ is given by

$$N^{mul-loc}(\rho^{AB}) = 2 \max\{0, -\lambda\}, \qquad (29)$$

where

$$\lambda = \frac{9(b - c)\gamma_A(\gamma_B - 1) + 2\gamma_B(1 - 9b + 3c) + 18b - 6c}{12}. \qquad (30)$$

It is easy to obtain the result that all the states, which are initially entangled ones, become separable if and only if $9(b - c)\gamma_A(\gamma_B - 1) + 2\gamma_B(1 - 9b + 3c) \geq 6(c - 3b)$.

(2) Qubit depolarizing channel only. The negativity for all the states which are initially entangled $(3b < c \leq$

$1 - 3b)$ is given by

$$N^{bit}(\rho^{AB}) = 2 \max\left\{0, \frac{2c - 6b - 3\gamma_A(c - b)}{4}\right\}. \qquad (31)$$

The states become separable for all values of $\gamma_A \geq \frac{2c - 6b}{3(c - b)}$.

(3) Qutrit depolarizing channel only. The negativity for all the states which are initially entangled $(3b < c \leq 1 - 3b)$ can be written as

$$N^{trit}(\rho^{AB}) = 2 \max\{0, \frac{3c - 9b - (1 - 9b + 3c)\gamma_B}{6}\}. \qquad (32)$$

The states become separable for all values of $\gamma_B \geq \frac{3c - 9b}{1 - 9b + 3c}$.

TABLE I: ESD in $2 \otimes 3$ systems under certain noises.

	multi-local	qubit noise only	qutrit noise only
dephasing	ESD with $b \neq 0$	ESD with $b \neq 0$	ESD with $b \neq 0$
phase-flip	ESD with $b \neq 0$	ESD with $b \neq 0$	ESD with $b \neq 0$
bit-(trit-) flip	exist ESD	ESD with $b \neq 0$	ESD
bit-(trit-) phase-flip	exist ESD	ESD with $b \neq 0$	ESD
depolarizing	exist ESD	ESD	ESD

IV. DISCUSSION AND SUMMARY

Putting all the pieces of our results together, one can see that ESD is a general phenomenon in a qubit-qutrit

system undergoing various independent noise channels

and we show the outcomes explicitly in Table.I. "ESD with $b \neq 0$" denotes ESD takes places in a qubit-qutrit system if and only if $b \neq 0$, that is, if b=0, no ESD occurs."exist ESD" denotes ESD may occurs in a qubit-qutrit system, but not necessary. "ESD" denotes ESD always occurs.

Decoherence which is characterized by the decay of the off-diagonal elements of the density matrix describing the system [24], results from the unwanted interactions of a quantum system with its environment. According to Eqs.(8,15,20,24,28), one can see that the coherence of a qubit-qutrit system can be destroyed completely when the system undergos the multi-local or local dephasing, phase-flip and depolarizing channels, while the disappearance of coherence does not occur even though $\gamma_A, \gamma_B \mapsto 1$ for multi-local or local the bit-(trit-)flip and bit-(trit-)phase-flip channels.

Using the negativity criterion for quantifying entanglement, it is shown that the density matrix of a hybrid qubit-qutrit system always suffers from ESD and decoherence under both local and multi-local depolarizing channels, which is different from the result that no ESD in any density matrix of a qubit-qutrit system occurs when only the qubit is coupled to its depolarizing channel presented in Ref. [14].

The entanglement dynamics of a specific one-parameter class of states interacting with a local and multi-lcoal dephasing noise in hybrid qubit-qutrit systems has been studied by Ann *et al.* [13] in 2008. We have generalized their study to a more general case, that is, a two-parameter class of states under the influence of local and multi-local dephasing, phase-flip, bit-(trit-) flip, bit-(trit-) phase-flip, and depolarizing channels. With analytical and numerical analysis, we have shown that ESD is a general phenomenon in a qubit-qutrit system under various noise channels, not only the case with dephasing and depolarizing ones [13]. It can only be avoided in some initial states undergoing particular noise channels.

In summary, our results show that the environment, which causes dephasing, phase-flip, bit-(trit-) flip, bit-(trit-) phase-flip, and depolarizing, affects the entanglement and the coherence of a hybrid qubit-qutrit system in a two-parameter class of entangled states in very different ways. ESD is a general phenomenon in a qubit-qutrit system undergoing various independent noise channels and it can only be avoided in some initial states undergoing particular noise channels. Moreover, we can divide those noise channels into two groups. For multi-local and local dephasing, phase-flip, and depolarizing noise channels, a time scale of disentanglement is usually shorter than the decay of the off-diagonal dynamics, and coherence disappears in an infinite-time limit $t \mapsto \propto$ in which $\gamma_A, \gamma_B \mapsto 1$. For multi-local or local bit-(trit-) flip and bit-(trit-) phase-flip channels, disentanglement may occur in the infinite-time limit, but the disappearance of coherence does not occur even though $\gamma_A, \gamma_B \mapsto 1$.

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